

8.2 - Operations with Polynomials

Name _____

In Exercises 1 - 8 determine whether the given algebraic expression is a polynomial. If it is, list its leading coefficient, constant term, and degree.

1. $1+x^3$ **yes**
 LC: 1 degree: 3
 constant: 1
2. $(7^x)+2x+1$ **NO**
3. $(x+\sqrt{3})(x-\sqrt{3})$ **yes**
 LC: 1 degree: 2
 constant: -3
4. $4x^2+3\sqrt{x}+5$ **NO**
5. $\frac{7}{x^2}+\frac{5}{x}-15$ **NO**
6. $(x-1)^k$ **yes**
 where k is a fixed, positive integer
 LC: 1 degree: k
 constant: 1 or -1

In Exercises 7 - 14 perform the indicated operations

7. $(m^2+3)-(4-3m)$
 $m^2+3-4+3m$
 m^2+3m-1
8. $(2x^2-4x+7)-(-2x^2+3x-7)$
 $2x^2-4x+7+2x^2-3x+7$
 $4x^2-7x+14$
9. $5a^4(a^2-4a+3)$
 $5a^6-20a^5+15a^4$
10. $(x+2)(x^2-4x+5)$
 $x^3-4x^2+5x+2x^2-8x+10$
 $x^3-2x^2-3x+10$
11. $(7x-3)^2 = (7x-3)(7x-3)$
 $49x^2-21x-21x+9$
 $49x^2-42x+9$
12. $(5-2x)^2 = (5-2x)(5-2x)$
 $25-10x-10x+4x^2$
 $25-20x+4x^2$
13. $(2x+5)(2x-5)-(2x+5)^2$
 $(4x^2-10x+10x-25)-(4x^2+10x+10x+25)$
 $4x^2-25-4x^2-20x-25$
 $-20x-50$
14. $(x+3)^2+(x-3)^2$
 $(x+3)(x+3)+(x-3)(x-3)$
 $x^2+6x+9+x^2-6x+9$
 $2x^2+18$

In Exercises 15 - 19, use synthetic division to find the quotient and remainder.

15. $(3x^4-8x^3+9x+5)\div(x-2)$

$$\begin{array}{r|rrrrr} 2 & 3 & -8 & 0 & 9 & 5 \\ & \downarrow & 6 & -4 & -8 & 2 \\ \hline & 3 & -2 & -4 & 1 & 7 \end{array}$$
 R: 7
 Q: $3x^3-2x^2-4x+1$
16. $(4x^3-3x^2+x+7)\div(x-2)$

$$\begin{array}{r|rrrr} 2 & 4 & -3 & 1 & 7 \\ & \downarrow & 8 & 10 & 22 \\ \hline & 4 & 5 & 11 & 29 \end{array}$$
 R: 29
 Q: $4x^2+5x+11$
17. $(3x^3-2x^2-8)\div(x+5)$

$$\begin{array}{r|rrrr} -5 & 3 & -2 & 0 & -8 \\ & \downarrow & -15 & 85 & -429 \\ \hline & 3 & -17 & 85 & -433 \end{array}$$
 R: -433
 Q: $3x^2-17x+85$
18. $(2x^4+5x^3-2x-8)\div(x+3)$

$$\begin{array}{r|rrrrr} -3 & 2 & 5 & 0 & -2 & -8 \\ & \downarrow & -6 & 3 & -9 & -33 \\ \hline & 2 & -1 & 3 & -11 & -41 \end{array}$$
 R: -41
 Q: $2x^3-x^2+3x-11$
19. $(x^6-x^5+x^4-x^3+x^2-x+1)\div(x-1)$

$$\begin{array}{r|rrrrrrr} 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ & \downarrow & -1 & 2 & -1 & 2 & -1 & 2 \\ \hline & 1 & -2 & 1 & -2 & 1 & -2 & 1 \end{array}$$
 R: -1
 Q: $x^5-2x^4+x^3-2x^2+x-2$

In Exercises 20 - 22, state the quotient and remainder when the first polynomial is divided by the second using long division. Check your division by calculating: (Divisor)(Quotient) + Remainder.

20. $3x^4+2x^2-6x+1; x+1$

$$\begin{array}{r} 3x^3-3x^2+5x-11 \text{ R } 12 \\ x+1 \overline{) 3x^4+0x^3+2x^2-6x+1} \\ \underline{-3x^4+3x^3} \\ -3x^3+2x^2 \\ \underline{+3x^3+3x^2} \\ -5x^2-6x \\ \underline{-5x^2+5x} \\ -11x+1 \\ \underline{+11x+11} \\ 12 \end{array}$$
21. $3x^4-3x^3-11x^2+6x-1; x^3+x^2-2$

$$\begin{array}{r} 3x-6 \\ x^3+x^2+0x-2 \overline{) 3x^4-3x^3-11x^2+6x-1} \\ \underline{-3x^4+3x^3+0x^2+6x} \\ -6x^3-11x^2+12x-1 \\ \underline{+6x^3+6x^2+0x+12} \\ -5x^2+12x-13 \end{array}$$
 $3x-6 \text{ R } -5x^2+12x-13$
22. $x^5-1; x-1$

$$\begin{array}{r} x^4+x^3+x^2+x+1 \text{ R } 0 \\ x-1 \overline{) x^5+0x^4+0x^3+0x^2+0x-1} \\ \underline{-x^5+x^4} \\ x^4+0x^3 \\ \underline{-x^4+x^3} \\ x^3+0x^2 \\ \underline{-x^3+x^2} \\ x^2+0x \\ \underline{-x^2+x} \\ x-1 \\ \underline{-x+1} \\ 0 \end{array}$$