

#### 8.4 - Remainder and Factor Theorems

Name \_\_\_\_\_

Find the zeros of the given polynomials algebraically by factoring and setting each factor equal to zero. Check your answers with your calculator.

1.  $f(x) = x^2 + 7x$

$$\begin{array}{l} x(x+7) \\ \hline x=0 \\ x+7=0 \\ x=-7 \end{array}$$

2.  $g(x) = x^2 - 5x - 6$

$$\begin{array}{l} (x-6)(x+1) \\ \hline \{0, -7\} \end{array}$$

3.  $h(x) = 6x^2 + x - 1$

$$\begin{array}{l} \cancel{6}x^2 \cancel{(6x^2+3x)}(2x-1) \\ \cancel{3}x \cancel{(2x+1)}-1(2x+1) \\ (3x-1)(2x+1) \quad \left(\frac{1}{2}, \frac{1}{3}\right) \\ \hline \{0, -1\} \end{array}$$

4.  $f(a) = (a+3)^2 - 64$

$$\begin{array}{l} (a+3+8)(a+3-8) \\ (a+11)(a-5) \\ \hline \{-11, 5\} \end{array}$$

5.  $g(t) = 3t^3 - 27t$

$$\begin{array}{l} 3t(t^2-9) \\ 3t(t-3)(t+3) \\ \hline \{0, -3, 3\} \end{array}$$

6.  $k(y) = 9y^4 + 3y^3 - 6y^2$

$$\begin{array}{l} 3y^2(3y^2 + y - 2) \\ 3y^2(y+1)(3y-2) \\ \hline \{0, -1, \frac{2}{3}\} \end{array}$$

7.  $h(x) = x^3 - 5x^2 - x + 5$

$$\begin{array}{l} x^2(x-5) - 1(x-5) \\ (x-5)(x^2-1) \\ (x-5)(x+1)(x-1) \\ \hline \{5, -1, 1\} \end{array}$$

8.  $f(x) = 3x^3 + 4x^2 - 27x - 36$

$$\begin{array}{l} (x^2-9)(3x+4) \\ (x-3)(x+3)(3x+4) \\ \hline \{3, -3, -4\} \end{array}$$

9.  $k(x) = 8x^3 - 12x^2 - 6x + 9$

$$\begin{array}{l} (2x-3)(4x^2-3) \\ 4x^2-3=0 \\ x=\pm\sqrt{\frac{3}{4}} \\ 4x^2-3=0 \\ \hline \left\{ \frac{3}{2}, \pm\frac{\sqrt{3}}{2} \right\} \end{array}$$

Find the remainder when  $f(x)$  is divided by  $g(x)$ , without using division.

10.  $f(x) = x^{10} + x^8$

$g(x) = x-1$

$$f(1) = 1^{10} + 1^8 = 2$$

11.  $f(x) = x^6 - 10$

$g(x) = x-2$

$$f(2) = 2^6 - 10 = 54$$

12.  $f(x) = 3x^4 - 6x^3 + 2x - 1$

$g(x) = x+1$

$$f(-1) = 3(-1)^4 - 6(-1)^3 + 2(-1) - 1 = 6$$

13.  $f(x) = x^3 - 2x^2 + 5x - 4$

$g(x) = x+2$

$$f(-2) = (-2)^3 - 2(-2)^2 + 5(-2) - 4 = -30$$

Use the Factor Theorem to determine whether  $h(x)$  is a factor of  $f(x)$ .

14.  $h(x) = x-1$

$f(x) = x^5 + 1$

$$f(1) = 1^5 + 1$$

NO

$$f(1) = 2 \leftarrow \text{should be zero}$$

$$f\left(\frac{1}{2}\right) = 0$$

☺

$$f(x) = 2x^4 + x^3 + x - \frac{3}{4}$$

$$f\left(\frac{1}{2}\right) = 0$$

YES

16.  $h(x) = x+2$

$f(x) = x^3 - 3x^2 - 4x - 12$

$$f(-2) = (-2)^3 - 3(-2)^2 - 4(-2) - 12$$

$$f(-2) = -24$$

NO

17.  $h(x) = x+1$

$f(x) = 14x^{99} + 65x^{56} - 51$

$$f(-1) = 14(-1)^{99} + 65(-1)^{56} - 51$$

$$f(-1) = -130$$

NO

Find a polynomial with the given degree,  $n$ , the given zeros, and no other zeros.

18.  $n=3$ ; zeros 1, 7, -4

$$(x-1)(x-7)(x+4)$$

19.  $n=3$ ; zeros 1, -1

$$(x-1)(x+1)^2$$

20.  $n=5$ ; zero 2

$$(x-2)^5$$

21. Find a polynomial function  $g$  of degree 3 such that  $g(10)=17$  and the zeros of  $g(x)$  are 0, 5, and 8.

(10, 17)

$g(0)=0$

$g(5)=0$

$g(8)=0$

$g(x) = x(x-5)(x-8) + c$

$$17 = 10(10-5)(10-8) + c$$

$$17 = 100 + c$$

$$c = -83$$

$$g(x) = x(x-5)(x-8) - 83$$

22. Find a polynomial function  $f$  of degree 4 such that  $f(3)=288$  and the zeros of  $f(x)$  are 0, -1, 2, and -3.

(3, 288)

$f(x) = x(x+1)(x-2)(x+3) + c$

$$288 = 3(3+1)(3-2)(3+3) + c$$

$$288 = 72 + c$$

$$-72 = -72$$

$$216 = c$$

$$f(x) = x(x+1)(x-2)(x+3) + 216$$

23. Find a number  $k$  such that  $x+2$  is a factor of  $x^3 + 3x^2 + kx - 2$ .

remainder = 0

$f(-2)=0$

$$(-2)^3 + 3(-2)^2 + k(-2) - 2 = 0$$

$$-8 + 12 - 2k - 2 = 0$$

$$-2k = -2$$

K=1

24. Find a number  $k$  such that  $x-1$  is a factor of  $k^2x^4 - 2kx^2 + 1$ .

$f(1)=0$

$$k^2(1)^4 - 2k(1)^2 + 1 = 0$$

$$k^2 - 2k + 1 = 0$$

$$(k-1)(k-1) = 0$$

$$k-1 = 0$$

K=1

25. When  $x^3 + cx + 4$  is divided by  $x+2$ , the remainder is 4. Find  $c$ .

$f(-2)=4$

$$(-2)^3 + c(-2) + 4 = 4$$

$$-8 - 2c = 0$$

$$-2c = 8$$

$$c = -4$$