

9.3 Graphing Rational Functions (Slant Asymptotes)

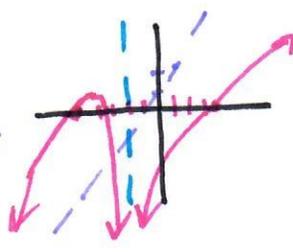
Name _____

Use algebra to determine the location of the vertical asymptotes, horizontal asymptotes / slant asymptotes and any holes in the graph of the function. Then sketch the graph.

$$\begin{array}{r} x+1 \\ x^2+4x+4 \end{array} \left| \begin{array}{r} x^3+5x^2-10x-50 \\ x^3+4x^2+4x \\ \hline x^2+4x+4 \\ x^2+4x+4 \\ \hline 0 \end{array} \right.$$

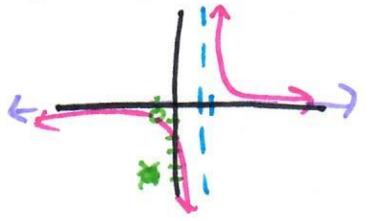
1. $f(x) = \frac{x^3 + 5x^2 - 10x - 50}{x^2 + 4x + 4} = \frac{(x+5)(x^2-10)}{(x+2)(x+2)}$

hole: none
 VA: $x = -2$
 HA: high none
 low none
 slant: $y = x + 1$
 ROOTS: $-5, \pm\sqrt{10}$



$$f(x) = \frac{x+1}{3x^2+x-2} = \frac{x+1}{(x+1)(3x-2)} = \frac{1}{3x-2}$$

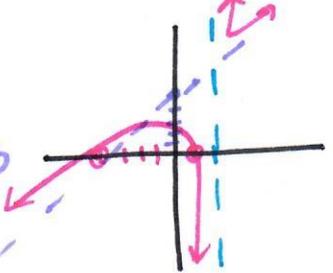
hole: $(-1, -\frac{1}{5})$
 VA: $x = \frac{2}{3}$
 HA: low high $y = 0$
 NO slant
 ROOTS: none



$$\begin{array}{r} 2 \mid 13-4 \\ \downarrow 2 \downarrow 10 \\ \hline 1 \mid 5 \mid 6 \end{array}$$

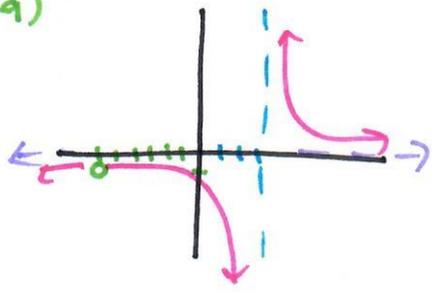
3. $f(x) = \frac{x^2 + 3x - 4}{x - 2} = \frac{(x+4)(x-1)}{x-2}$

hole: none
 VA: $x = 2$
 HA: none
 slant: $y = x + 5$
 ROOTS: $-4, 1$



4. $f(x) = \frac{x+6}{x^2+3x-18} = \frac{x+6}{(x+6)(x-3)} = \frac{1}{x-3}$

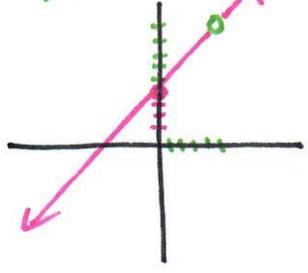
hole: $(-6, \frac{1}{9})$
 VA: $x = 3$
 HA: $y = 0$
 NO slant
 ROOTS: none



$$\begin{array}{r} 5 \mid 1-1-20 \\ \downarrow 5 \downarrow 20 \\ \hline 1 \mid 4 \mid 0 \end{array}$$

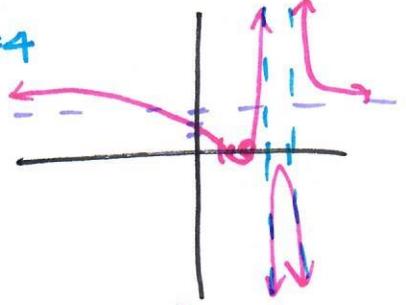
5. $f(x) = \frac{x^2 - x - 20}{x - 5} = \frac{(x-5)(x+4)}{x-5} = x+4$

hole: $(5, 9)$
 VA: none
 HA: none
 line with a hole



6. $f(x) = \frac{3x^2 - 11x + 10}{x^2 - 7x + 12} = \frac{(x-2)(3x-5)}{(x-3)(x-4)}$

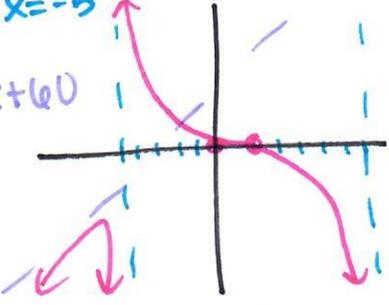
hole: none
 VA: $x = 3$ and $x = 4$
 HA: $y = 3$
 no slant
 ROOTS: $2, \frac{5}{3}$



$$\begin{array}{r} 2x+40 \\ x^2-x-30 \end{array} \left| \begin{array}{r} 2x^3-2x^2+10x+0 \\ 2x^3-2x^2-60x \\ \hline 60x+0 \end{array} \right.$$

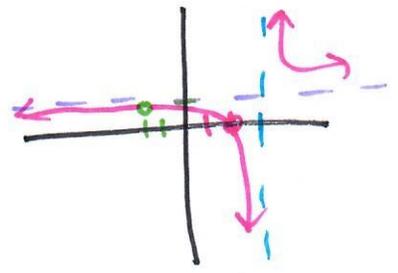
$f(x) = \frac{2x^3 - 2x^2}{x^2 - x - 30} = \frac{2x^2(x-1)}{(x-6)(x+5)}$

hole: none
 VA: $x = 6$ and $x = -5$
 HA: none
 slant: $y = 2x + 60$
 ROOTS: $0, 1$



8. $f(x) = \frac{x^2 - 4}{x^2 - x - 6} = \frac{(x-2)(x+2)}{(x-3)(x+2)} = \frac{x-2}{x-3}$

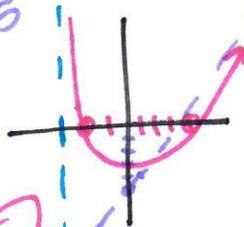
hole: $(-2, \frac{4}{5})$
 VA: $x = 3$
 HA: $y = 1$
 NO slant
 ROOTS: 2



9. $f(x) = \frac{x^2 - 2x - 8}{x + 3} = \frac{(x-4)(x+2)}{x+3}$

hole: none
 VA: $x = -3$
 HA: none
 slant: $y = x - 5$

Roots: 4, -2

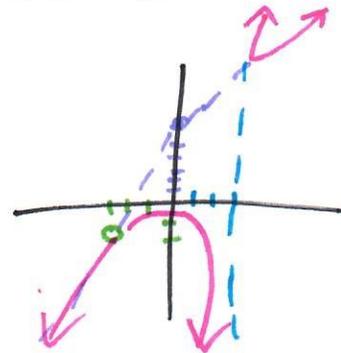


$$\begin{array}{r} -3 \overline{) 1 - 2 - 8} \\ \underline{-3 5} \\ 1 - 5 1 \end{array}$$

10. $f(x) = \frac{x^3 + 27}{x^2 - 9} = \frac{(x+3)(x^2 + 3x + 9)}{(x-3)(x+3)}$

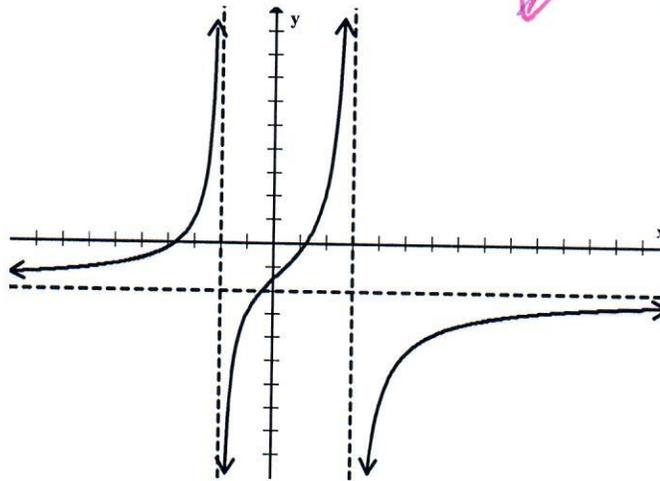
hole: $(-3, -3/2)$
 VA: $x = 3$
 HA: none
 slant: $y = x + 6$

roots: none



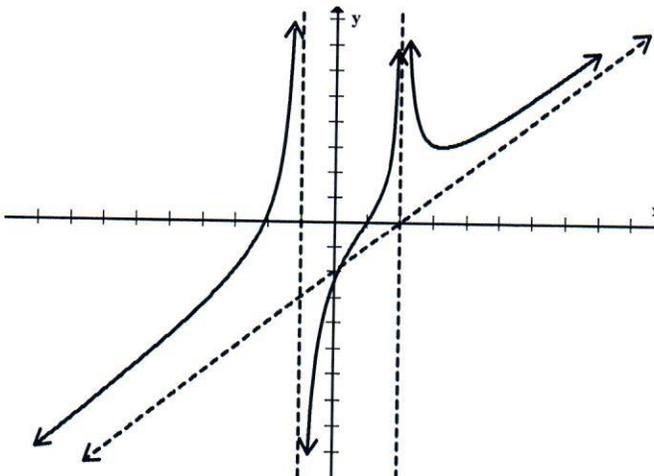
11. Use the graph to determine:

- Right as $x \rightarrow -2^+$, $f(x) \rightarrow -\infty$
 Left as $x \rightarrow -2^-$, $f(x) \rightarrow \infty$
 as $x \rightarrow 3^+$, $f(x) \rightarrow -\infty$
 as $x \rightarrow 3^-$, $f(x) \rightarrow \infty$
 as $x \rightarrow \infty$, $f(x) \rightarrow -2$
 as $x \rightarrow -\infty$, $f(x) \rightarrow -2$



12. Use the graph to determine:

- as $x \rightarrow -1^+$, $f(x) \rightarrow -\infty$
 as $x \rightarrow -1^-$, $f(x) \rightarrow \infty$
 as $x \rightarrow 2^+$, $f(x) \rightarrow \infty$
 as $x \rightarrow 2^-$, $f(x) \rightarrow \infty$
 as $x \rightarrow \infty$, $f(x) \rightarrow \infty$
 as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$



For #13-14, Write the equation of the function with the given characteristics.

13. hole at $x = -5$ ($x+5$) cancels
 VA at $x = 2$ and $x = -4$ ($x-2$) and ($x+4$) bottom
 x-intercepts at $x = -6$ and $x = 3$ ($x+6$) ($x-3$) top
 HA at $y = 3$ same degree

$$y = \frac{3(x+5)(x+6)(x-3)}{(x+5)(x-2)(x+4)}$$

14. VA at $x = 1$ and $x = 3$
 has a slant asymptote one degree higher
 only x-intercepts are $x = 2$ and $x = 5$

$$y = \frac{(x-2)(x-5)^2}{(x-1)(x-3)}$$