

9.3 Graphing Rational Functions

Slant Asymptotes

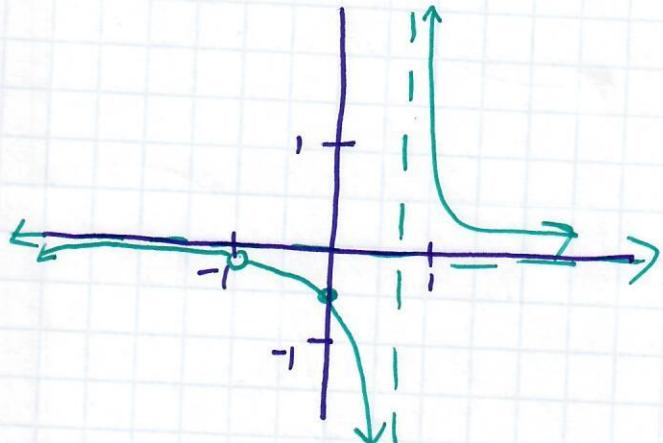
$$\textcircled{1} \quad f(x) = \frac{x+1}{3x^2+x-2} = \frac{x+1}{(x+1)(3x-2)}$$

$$3 \cancel{\times} \begin{matrix} -6 \\ 1 \\ -2 \end{matrix} \quad \begin{matrix} (3x^2+3x)(2x-2) \\ 3x(x+1)-2(x+1) \\ (x+1)(3x-2) \end{matrix} \quad f(x) = \frac{1}{3x-2}$$

$$\text{RD: } x+1=0 \quad x=-1 \quad \left(-1, -\frac{1}{5}\right) \quad \text{VA: } 3x-2=0 \quad x=\frac{2}{3}$$

$$y = \frac{1}{3(-1)-2} = \frac{1}{-3-2} = -\frac{1}{5} \quad \text{HA: } \frac{\text{low}}{\text{high}} \quad y=0$$

x-int: none
y-int: $(0, -\frac{1}{2})$



$$\textcircled{2} \quad f(x) = \frac{x^2-2x-8}{x+3} = \frac{(x-4)(x+2)}{x+3}$$

$$\text{VA: } x=-3$$

$$\text{HA: } \frac{\text{high}}{\text{low}} \text{ none}$$

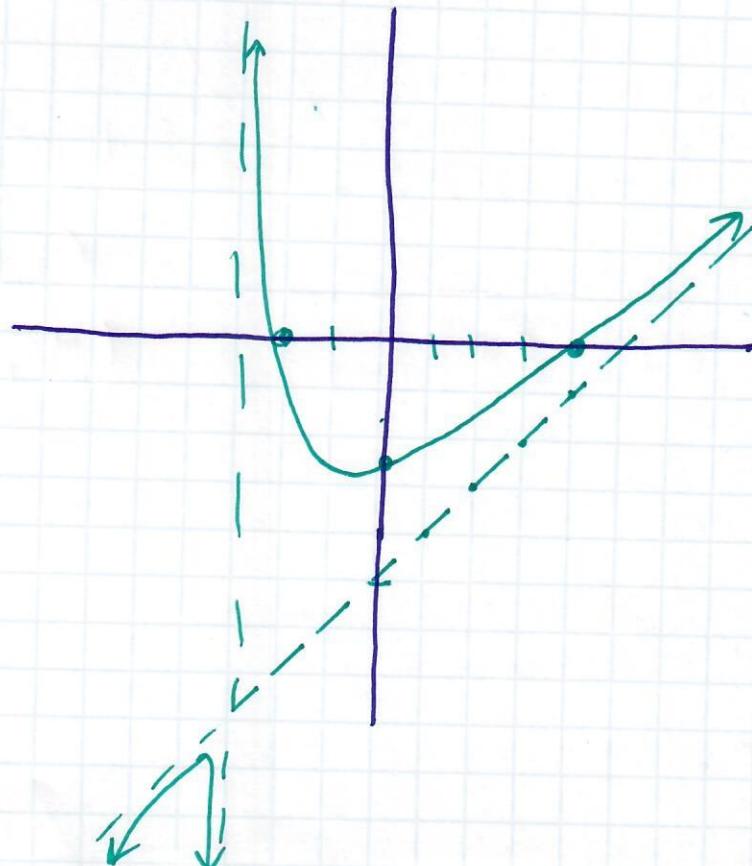
$$\text{SA: } \frac{-3|1-2-8}{\downarrow -3 \quad 15} \quad \begin{matrix} 1 \\ -3 \\ 15 \\ 1-5 \\ 7 \end{matrix}$$

$$y = x - 5$$

$$\text{x-int: } (4, 0), (-2, 0)$$

$$\text{y-int: } (0, -8/3)$$

no hole



$$\textcircled{3} \quad f(x) = \frac{x^2 + 3x - 4}{x-2} = \frac{(x+4)(x-1)}{x-2}$$

no hole

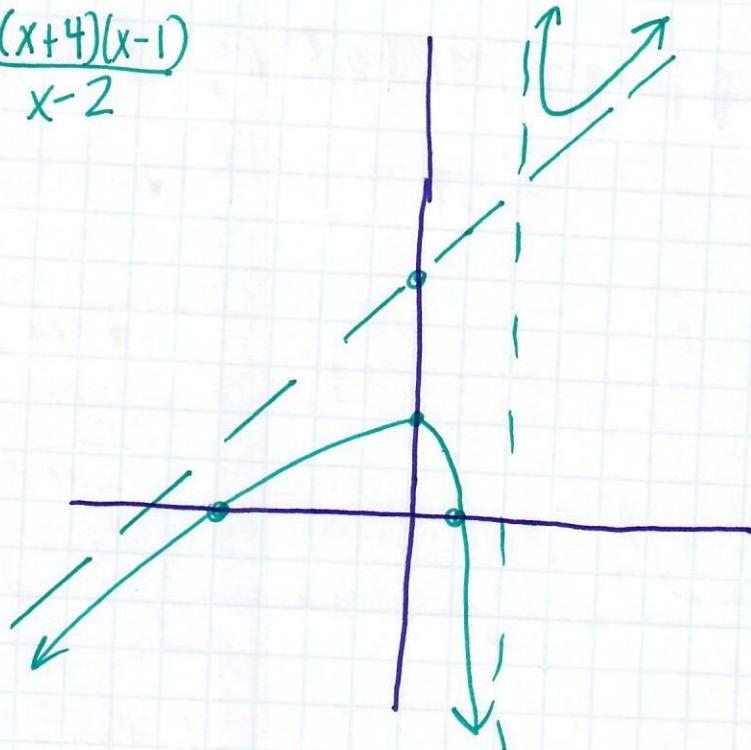
VA: $x=2$

HA: none

↳ SA: $\begin{array}{r} 2 | 1 & 3 & -4 \\ & \downarrow 2 & 10 \\ & 1 & 5 & 6 \end{array}$
 $y = x + 5$

x-int: $(-4, 0) \nexists (1, 0)$

y-int: $(0, 2)$



$$\textcircled{4} \quad f(x) = \frac{x+6}{x^2 + 3x - 18} = \frac{x+6}{(x+6)(x-3)}$$

RD: $x+6=0$ $x=-6$ $(-6, -\frac{1}{9})$ $f(x) = \frac{1}{x-3}$

$y = \frac{1}{-6-3} = -\frac{1}{9}$

VA: $x=3$
HA: $y=0$

x-int: none
y-int: $(0, -\frac{1}{3})$

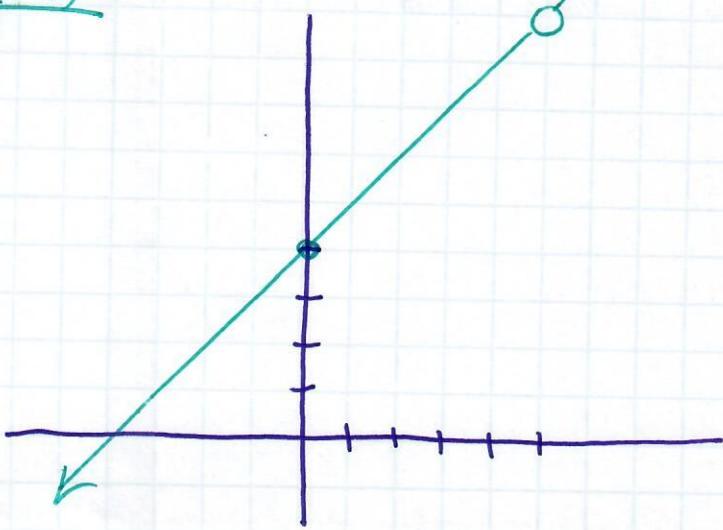


$$\textcircled{5} \quad f(x) = \frac{x^2 - x - 20}{x-5} = \frac{(x-5)(x+4)}{x-5}$$

Line w/ hole

RD: $x-5=0$ $x=5$ $(5, 9)$

$y = 5+4 = 9$



$$\textcircled{6} \quad f(x) = \frac{x^3 + 27}{x^2 - 9} = \frac{(x+3)(x^2 - 3x + 9)}{(x+3)(x-3)}$$

RD: $x+3=0$
 $x=-3$ $(-3, -\frac{9}{2})$

$$y = \frac{(-3)^2 - 3(-3) + 9}{-3-3} = \frac{27}{-6}$$

$$\frac{x^2 - 3x + 9}{x-3}$$

VA: $x=3$

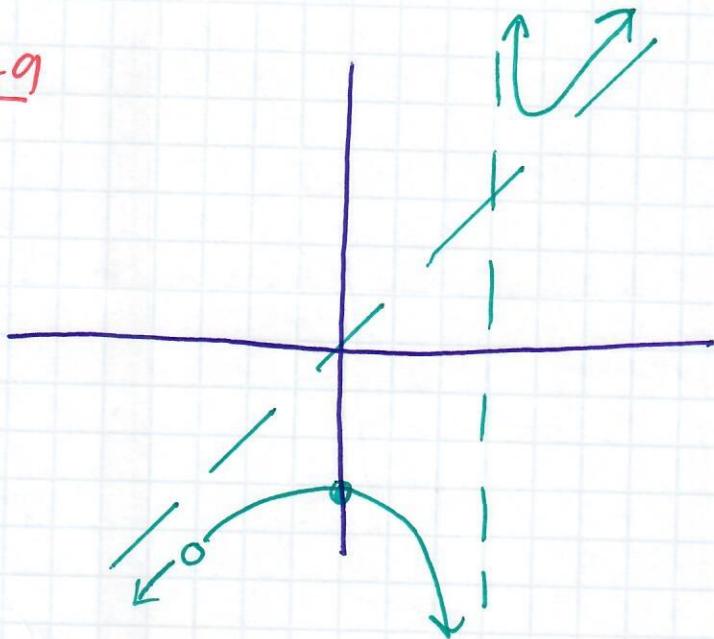
HA: none

SA: $\begin{array}{r} 3 \\ \hline 1 & -3 & 9 \\ \downarrow & 3 & 0 \\ 1 & 0 & 9 \end{array}$

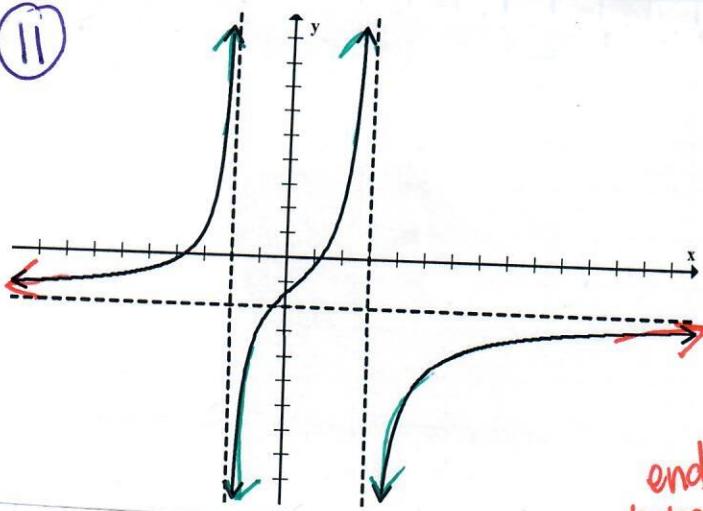
$$y=x$$

x-int: none

$$y\text{-int: } (0, -3)$$



\textcircled{11}



as $x \rightarrow -2^+$, $f(x) \rightarrow -\infty$
 from right

as $x \rightarrow -2^-$, $f(x) \rightarrow \infty$
 from left

as $x \rightarrow 2^+$, $f(x) \rightarrow -\infty$
 right

as $x \rightarrow 2^-$, $f(x) \rightarrow \infty$
 left

end behaviors
 $\left\{ \begin{array}{l} \text{as } x \rightarrow \infty, f(x) \rightarrow -2 \\ \text{as } x \rightarrow -\infty, f(x) \rightarrow -2 \end{array} \right.$

as $x \rightarrow -1^+$, $f(x) \rightarrow -\infty$
 right

as $x \rightarrow -1^-$, $f(x) \rightarrow \infty$
 left

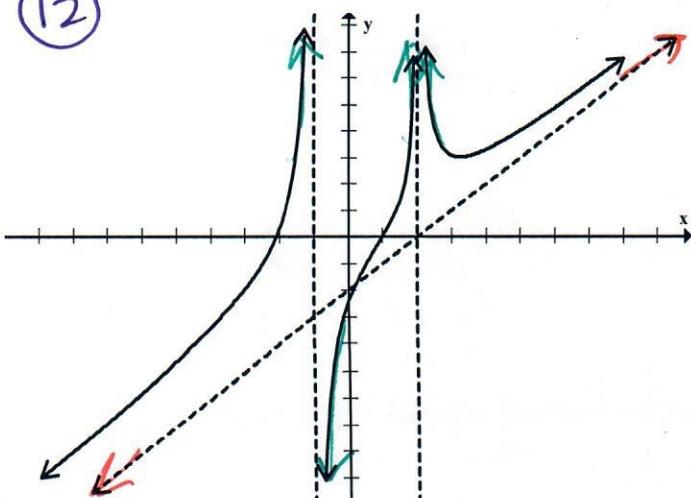
as $x \rightarrow 2^+$, $f(x) \rightarrow \infty$
 right

as $x \rightarrow 2^-$, $f(x) \rightarrow \infty$
 left

as $x \rightarrow \infty, f(x) \rightarrow \infty$

as $x \rightarrow -\infty, f(x) \rightarrow -\infty$

\textcircled{12}



as $x \rightarrow \infty, f(x) \rightarrow \infty$

as $x \rightarrow -\infty, f(x) \rightarrow -\infty$

(13) hole at $x = -5$

$(x+5)$ will cancel (top & bottom)

VA at $x = 2$ and $x = -4$

bottom needs $(x-2)$ and $(x+4)$

x-int at $x = -6$ and $x = 3$

top needs $(x+6)$ and $(x-3)$

HA at $y = 3$

• same degree on top & bottom

• coefficient of top = 3

$$\text{ex. } f(x) = \frac{3(x+5)(x+6)(x-3)}{(x+5)(x-2)(x+4)}$$

Answers can vary

(14) VA at $x = 1$ & $x = 3$

bottom needs $(x-1)$ and $(x-3)$

has slant asymptote

top one degree higher than bottom

only x-int at $x = 2$ & $x = 5$

top needs $(x-2)$ and $(x-5)$

$$\text{ex. } f(x) = \frac{(x-2)(x-5)^2}{(x-1)(x-3)}$$

Answers
vary