Name: Key

Unit 10 Log Review

Remember to study your notes, homework, and quiz. The "Ultimate Log Worksheet" is also a GREAT review!

1. Convert the following into LOGARITHMIC form.

A. $2^9 = 512$

$\log_2 512 = 9$

B. $9^0 = 1$

$\log_9 1 = 0$

C. $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

D. $8^{\frac{1}{3}} = 2$

$\log_8 2 = \frac{1}{3}$

2. Convert the following into EXPONENTIAL form.

A. $\log_6 1296 = 4$

$6^4 = 1296$

B. $\log_7 7 = 1$

$7^1 = 7$

C. $\log_6 \frac{1}{36} = -2$

$6^{-2} = \frac{1}{36}$

D. $\log_{16} 4 = \frac{1}{2}$

$16^{\frac{1}{2}} = 4$

Solve each equation for x:

3. $\log_3 81 = x$

$3^x = 81$ $\boxed{x=4}$

4. $\log_{\sqrt{2}} x = -3$

$(\sqrt{2})^{-3} = x$ $\boxed{x = \frac{1}{2^{3/2}}}$

5. $\log_4 x = \frac{1}{2}$

$4^{1/2} = x$ $\boxed{2 = x}$

$\sqrt{4} = x$

6. $\log_x \frac{1}{32} = -5$

$x^{-5} = \frac{1}{32}$ $\boxed{x=2}$

7. $\log_5 (x-4) = 0$

$5^0 = x-4$

$1 = x-4$

$+4 \quad +4$

$\boxed{5=x}$

8. $\log_b b^4 = x$

$4\cancel{\log_b b} = x$

$\boxed{4=x}$

9. Use the laws of logarithms to expand the expression

A. $\log_2 (xy)^{10}$

$\log_2 x^{10}y^{10} = \log_2 x^{10} + \log_2 y^{10}$

$= \boxed{10\log_2 x + 10\log_2 y}$

B. $\log_a \left(\frac{x^2}{yz^3}\right)$

$\log_a x^2 - \log_a yz^3$

$2\log_a x - (\log_a y + \log_a z^3) =$

$2\log_a x - (\log_a y + 3\log_a z)$

$\boxed{2\log_a x - \log_a y - 3\log_a z}$

10. Use the Laws of Logarithms to combine the expression into a single log

A. $\log_5 (x^2-1) - \log_5 (x-1)$

$\log_5 \left(\frac{x^2-1}{x-1}\right) = \log_5 \left(\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\right)$

$= \boxed{\log_5 (x+1)}$

B. $\ln(a+b) + \ln(a-b) - 2\ln c$

$\ln\left(\frac{(a+b)(a-b)}{c^2}\right) = \boxed{\ln\left(\frac{a^2-b^2}{c^2}\right)}$

Solve for *x*: *leave answers as simplified fractions as needed*

11. $3^{2x-1} = 27$

$3^{2x-1} = 3^3$ $\boxed{x=2}$

$2x-1 = 3$

12. $3^x = \left(\frac{1}{3}\right)^{x-3}$

$3^x = (3^{-1})^{x-3}$ $\boxed{x = 3/2}$

$x = -x+3$

13. $\sqrt{9}^{5x-1} = \left(\frac{1}{81}\right)^{x-4}$

$(9^{\frac{1}{2}})^{5x-1} = (9^{-2})^{x-4}$

$\frac{1}{2}(5x-1) = -2(x-4)$

$\boxed{x = 17/9}$

14. $\log x = \frac{1}{3}\log 64 - \frac{1}{5}\log 32$

$\log x = \log\left(\frac{64^{1/3}}{32^{1/5}}\right)$

$\log x = \log\left(\frac{4}{2}\right)$

$\boxed{x=2}$

15. $\log_4 (x-4) + \log_4 (x+4) = 2\log_4 3$

$\log_4((x-4)(x+4)) = \log_4 3^2$

$x^2 - 16 = 9$

$x^2 = 25$

$\boxed{x = 5}$

16. $\log_2 x + \log_2 (x-6) = 4$

$\log_2(x^2-6x) = 4$

$2^4 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$x = 8 \quad \cancel{x=-2}$

$\boxed{\{8\}}$

Solve for x: *Round to 2 decimal places*

17. $\ln x = -4.2$

$\log_e x = -4.2$

$e^{-4.2} = x$

$\boxed{x = 0.01}$

18. $2.13^x = 6.3$

$\log_{2.13} 6.3 = x$

$\boxed{2.43 = x}$

19. $3e^{5x} = 12$

$\frac{3e^{5x}}{3} = \frac{12}{3}$

$e^{5x} = 4$

$\log_e 4 = 5x$

$\frac{\ln 4}{5} = x$

$\boxed{0.28 = x}$

20. $3\ln 5x = 27$

$\frac{3\ln 5x}{3} = \frac{27}{3}$

$\ln 5x = 9$

$\frac{e^9}{5} = \frac{5x}{5}$

$\boxed{x = 1{,}620.62}$

21. $3e^{-x} - 4 = 9$

$+4 \quad +4$

$\frac{3e^{-x}}{3} = \frac{13}{3}$

$e^{-x} = 13/3$

$-x = \ln(13/3)$

$\boxed{x = -1.47}$

22. $\ln(2x+7) = -3$

$e^{-3} = 2x + 7$

$-7 \qquad -7$

$\frac{e^{-3} - 7}{2} = x$

$\boxed{x = -3.48}$

23. Evaluate:

A. $49^{-\log_7 4}$

$(7^2)^{\log_7 4^{-1}}$

$4^{-2} = \boxed{\frac{1}{16}}$

B. $6^{3\log_6 2 + 2\log_6 3}$

$6^{\log_6 2^3 + \log_6 3^2}$

$6^{\log_6 (8 \cdot 9)}$

$\boxed{72}$

C. $\log_2(\log_2(\log_2 16))$

$\log_2(\log_2(4))$

$\log_2(2)$

$\boxed{1}$

D. $\log_{16}(\log_2(\log_3 9))$

$\log_{16}(\log_2(2))$

$\log_{16}(1)$

$\boxed{0}$

24. The half life of a certain substance is 18 days. If there are 8.3 grams initially, when will there be 0.5 grams left? *Round to 2 decimal places*

$\frac{1}{2} = e^{18r}$ $\quad r \approx -0.039$

$\frac{\ln(1/2)}{18} = r$

$0.5 = 8.3e^{rt}$

$\ln\left(\frac{0.5}{8.3} = e^{rt}\right)$

$\frac{\ln\left(\frac{0.5}{8.3}\right)}{-0.039} = t$

$\boxed{t = 72.96 \text{ days}}$

25. What is the total value after 7 years of an initial investment of \$5250 that earns interest at the rate of 6.1%, compounded monthly?

12

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 5250\left(1 + \frac{.061}{12}\right)^{12 \cdot 7}$

$\boxed{\$8037.73}$

26. A \$2500 investment earns interest compounded quarterly. Determine the interest rate needed in order for the money to grow to \$4000 over the course of 4 years. *Give the answer as a percentage rounded to 1 decimal.*

4

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$4000 = 2500\left(1 + \frac{r}{4}\right)^{4 \cdot 4}$

$\sqrt[16]{1.6} = \left(1 + \frac{r}{4}\right)^{16}$

$1.03 = 1 + \frac{r}{4}$

$r = .1192$

$\boxed{11.9\%}$

27. How long will it take an investment of \$1100 at 7.45% APR to grow to \$2500 if the interest rate is compounded monthly? *Round to 1 decimal place*

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$2500 = 1100\left(1 + \frac{.0745}{12}\right)^{12 \cdot t}$

$\boxed{11.1 \text{ years}}$

28. How long will it take an investment of \$3000 to double if it is invested in an account earning 4.75% interest compounded continuously?

$A = Pe^{rt}$

$6000 = 3000e^{.0475t}$

$\boxed{14.6 \text{ years}}$