

# Pre-AP Pre-Calculus Sequences and Series Test Review

1. Write the sum using sigma notation.

$$9 - 10x + 11x^2 - 12x^3 + \dots + 97x^{88}$$

Alternating

$$\sum_{n=9}^{97} (-1)^{n-1} n x^{n-8} \quad \text{or} \quad \sum_{n=0}^{88} (-1)^n (n+9) x^n$$

2. Find the sum.

$$\sum_{k=1}^4 k 2^k = 1(2)^1 + 2(2)^2 + 3(2)^3 + 4(2)^4 = \boxed{98}$$

3. Write the following sum using sigma notation.

$$1 - 5x^4 + 9x^8 - 13x^{12} + \dots + 121x^{120} \quad \begin{matrix} 1, 5, 9, 13, \dots \\ \hat{4}, \hat{4}, \hat{4}, \hat{4} \end{matrix} \quad 4n-3 \quad \begin{matrix} 0, 4, 8, 12, \dots \\ \hat{4}, \hat{4}, \hat{4} \end{matrix} \quad 4n-4$$

$$\sum_{n=1}^{31} (-1)^{n-1} (4n-3) x^{4n-4}$$

4. The first term of the arithmetic sequence  $a$  is 4 and common difference  $d$  is 6. Find the  $n$ th term and the 10th term.

$$a_1 = 4$$

$$d = 6$$

$$a_n = 4 + 6(n-1)$$

$$a_{10} = 4 + 6(10-1)$$

$$58$$

5. Find the first five terms and determine if the sequence is arithmetic. Adding/Subtracting

$$a_n = 2 + 6n$$

$$a_1 = 8 \quad a_3 = 20 \quad a_5 = 32$$

$$a_2 = 14 \quad a_4 = 26$$

Yes

6. Find the 60th term of the arithmetic sequence.

$$26.2, 29.9, 33.6, 37.3, \dots$$

$$a_n = 26.2 + 3.7(n-1)$$

$$a_{60} = 26.2 + 3.7(60-1)$$

$$\boxed{244.5}$$

7. Find the  $n$ th term of the arithmetic sequence.

$$2, 2+s, 2+2s, 2+3s, \dots$$

$$a_n = 2 + s(n-1)$$

$$+s \quad +s \quad +s$$

8. Find the 18th term of the arithmetic sequence.

$$-t, -t+3, -t+6, -t+9, \dots$$

$$a_n = -t + 3(n-1)$$

$$+3 \quad +3 \quad +3$$

$$a_{18} = -t + 3(18-1) = \boxed{-t + 51}$$

9. Which term of the arithmetic sequence 3, 8, 13, ... is 73?

$$a = 3 \\ d = 5 \\ a_n = 3 + 5(n-1)$$

+5 +5

$$73 = 3 + 5(n-1) \\ 73 = 3 + 5n - 5 \\ 75 = 5n$$

$$\boxed{n = 15}$$

10. A partial sum of an arithmetic sequence is given. Find the sum.

$$3 + 7 + 11 + \dots + 39$$

$$d = 4$$

$$39 = 3 + 4(n-1)$$

$$a = 3$$

$$39 = 3 + 4n - 4$$

$$40 = 4n$$

$$n = 10$$

$$S_n = n \left( \frac{a + a_n}{2} \right) = 10 \left( \frac{3 + 39}{2} \right) = \boxed{210}$$

11. Determine whether the sequence

$$6, 24, 96, 384, \dots$$

$$\times 4 \quad \times 4 \quad \times 4$$

is geometric. If it is geometric, find the common ratio.

$$\text{Yes, } r = 4$$

12. Determine the common ratio, the 6th term, and the  $n$ th term of the geometric sequence.

$$4, 12, 36, 108, \dots \quad a_n = 4(3)^{n-1}$$

$$r = \frac{12}{4} = 3 \quad a_6 = 4(3)^{6-1} = \boxed{972}$$

13. Determine the  $n$ th term of the geometric sequence.

$$x, \frac{x^2}{5}, \frac{x^3}{25}, \frac{x^4}{125}, \dots \quad r = \frac{x}{5} \quad a = x$$

$$a_n = x \left( \frac{x}{5} \right)^{n-1}$$

14. Which term of the geometric sequence 5, 20, 80, ... is 20480?

$$a = 5$$

$$r = 4$$

$\times 4 \quad \times 4$

$$20480 = 5(4)^{n-1}$$

$$6 = n-1$$

$$4096 = 4^{n-1}$$

$$\boxed{n=7}$$

$$4^6 = 4^{n-1}$$

15. Find the sum. geometric

$$a = 1 \quad 1 + 4 + 16 + \dots + 4096 \quad S_n = a \left( \frac{1-r^n}{1-r} \right) = 1 \left( \frac{1-4^7}{1-4} \right) = \boxed{5461}$$

$$r = 4$$

$$n = 7$$

16. Find the sum of the infinite geometric series.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

since  $|r| < 1$ , series converges

$$a = 1 \quad r = \frac{1}{3}$$

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \boxed{\frac{3}{2}}$$

17. Use the Binomial Theorem to expand the expression  $(3-x)^5$ .

$$\begin{aligned} & \binom{5}{0} 3^5 (-x)^0 + \binom{5}{1} 3^4 (-x)^1 + \binom{5}{2} 3^3 (-x)^2 + \binom{5}{3} 3^2 (-x)^3 + \binom{5}{4} 3^1 (-x)^4 + \binom{5}{5} 3^0 (-x)^5 \\ & 1 \cdot 243 + 5 \cdot 81 (-x) + 10 \cdot 27 x^2 + 10 \cdot 9 (-x^3) + 5 \cdot 3 (x^4) + 1 \cdot 1 (-x^5) \end{aligned}$$

$$\boxed{243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5}$$

18. Find the first three terms in the expansion of  $(x+2y)^{15}$ .

$$\binom{15}{0}(x)^{15}(2y)^0 + \binom{15}{1}(x)^{14}(2y)^1 + \binom{15}{2}(x)^{13}(2y)^2$$

$$15x^{15} \cdot 2y$$

$$105x^{13} \cdot 4y^2$$

$$x^{15} + 30x^{14}y + 420x^{13}y^2$$

$$\frac{15!}{13!2!} = \frac{15 \cdot 14}{2} = 105$$

19. Find the middle term in the expansion of  $(x^4 + 1)^{20}$ .

21 terms total

middle = 11

$$\binom{20}{10}(x^4)^{10}(1)^{10}$$

so  $k=10$

$$184756x^{40}$$

$$\frac{20}{10!10!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 184756$$

20. Find the term containing  $x^6$  in the expansion of  $(x+2y)^{10}$ .

$$\frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$$

$$\binom{10}{4}(x)^6(2y)^4 = 210x^6(2^4)y^4 = 3360x^6y^4$$

21. Find the sum.

$$\sum_{k=2}^6 2^{k-2} = \boxed{31} \quad 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

22. The 12th term of an arithmetic sequence is 34, and the fifth term is 20. Find the 20th term.

$$a_{12} = 34 \quad a_5 = 20 \quad d = \frac{34 - 20}{12 - 5} = 2 \quad 20 = a_1 + 2(5-1) \quad a_{20} = 12 + 2(20-1)$$

$$12 = a_1$$

23. The first term of an arithmetic sequence is 3, and the common difference is 2. Is 5,981 a term of this sequence? If so, which term is it?

$$a = 3$$

$$d = 2$$

$$5981 = 3 + 2(n-1)$$

$$5981 = 3 + 2n - 2$$

$$2990 = n$$

yes (n needed to be an integer)

24. A partial sum of an arithmetic sequence is given. Find the sum.

$$\begin{array}{ccccccc} +.3 & +.3 & +.3 \\ -30 & -29.7 & -29.4 & \dots & -0.3 \end{array}$$

$$-0.3 = -30 + 0.3(n-1)$$

$$29.7 = 0.3n - 0.3$$

$$S = -1515$$

$$n = 100$$

$$S_{100} = 100 \left( \frac{-30 + (-0.3)}{2} \right)$$

25. A partial sum of an arithmetic sequence is given. Find the sum.

$$\sum_{n=0}^{20} (1-7n) \quad \text{1st term } 1-7(0) = 1$$

$$S_{21} = 21 \left( \frac{1 + (-139)}{2} \right)$$

$$\text{last term } 1-7(20) = -139$$

$$S = -1449$$

21 terms total

26. An arithmetic sequence has first term  $a_1 = 7$  and fourth term  $a_4 = 22$ . How many terms of this sequence must be added to get 3,402?  $d = \frac{22-7}{4-1} = 5$

$$a_n = 3402 = \frac{n}{2}(2(7) + 5(n-1))$$

$$2 \left( 3402 = \frac{n}{2} (14 + 5n - 5) \right)$$

$$6804 = 5n^2 + 9n$$

$$n = 36$$

27. Determine whether the sequence is geometric. If it is geometric, find the common ratio.

$$1.0, 1.3, 1.69, 2.197, \dots \quad r = \frac{1.3}{1.0} = 1.3 \stackrel{?}{=} \frac{1.69}{1.3} = 1.3 \quad (\boxed{r=1.3})$$

yes

28. Which term of the geometric sequence 2, 4, 8, ... is 4,096?

12 th term

$$\begin{aligned} a &= 2 \\ r &= 2 \end{aligned}$$

$$\begin{aligned} 4096 &= 2(2)^{n-1} \\ 2048 &= 2^{n-1} \quad n = 12 \\ 2^11 &= 2^{n-1} \end{aligned}$$

29. The second and the fifth terms of a geometric sequence are 4 and 32, respectively. Is 512 a term of this sequence? If so, which term is it?

$$\begin{aligned} a_2 &= 4 \\ a_5 &= 32 \end{aligned} \quad r = 2$$

$$\begin{aligned} 512 &= 2(2)^{n-1} \\ 256 &= 2^{n-1} \\ 2^8 &= 2^{n-1} \end{aligned}$$

$$\boxed{n=9}$$

30. Find the sum of the infinite geometric series.

$$4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots \quad r = \frac{1}{3} \quad a = 4 \quad S = \frac{4}{1 - \frac{1}{3}} = \boxed{6}$$

31. Find the sum.

$$\sum_{k=1}^5 \frac{11}{k} = \frac{1507}{60}$$

$$\frac{11}{1} + \frac{11}{2} + \frac{11}{3} + \frac{11}{4} + \frac{11}{5}$$

32. Write the sum using sigma notation.

$$4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40$$

+4 +4 +4

$$\sum_{n=0}^9 4n + 4 \quad \text{or} \quad \sum_{n=1}^{10} 4n$$

33. Determine whether the sequence is arithmetic. If it is arithmetic, find the common difference.

$$5, 11, 13, 23, \dots \quad \begin{matrix} +6 \\ +2 \end{matrix} \quad \text{NO}$$

34. Find the sum.

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{1}{1,024}$$

$$a = 1 \quad r = -\frac{1}{2}$$

$$\begin{aligned} \frac{1}{1024} &= 1 \left(-\frac{1}{2}\right)^{n-1} \\ \left(-\frac{1}{2}\right)^{10} &= \left(-\frac{1}{2}\right)^{n-1} \end{aligned} \quad S_{11} = 1 \left( \frac{1 - \left(-\frac{1}{2}\right)^{11}}{1 - \left(-\frac{1}{2}\right)} \right) = \boxed{\frac{683}{1024}}$$

$$n = 11$$

35. Find the sum of the infinite geometric series.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

$$r = \frac{1}{3}$$

$$S = \frac{1}{1 - \frac{1}{3}} = \boxed{\frac{3}{2}}$$

36. Find the sum of the infinite geometric series.

$$\frac{2}{7} - \frac{8}{49} + \frac{32}{343} - \dots$$

$$r = -\frac{4}{7}$$

$$|r| < 1$$

$$S = \frac{\frac{2}{7}}{1 - \left(-\frac{4}{7}\right)} = \boxed{\frac{2}{11}}$$