Properties of Trigonometric Functions

Reciprocal

$$\cot x = \frac{1}{\tan x} \quad \text{or} \quad \tan x \cot x = 1$$

$$\sec x = \frac{1}{\cos x} \quad \text{or} \quad \cos x \sec x = 1$$

$$\csc x = \frac{1}{\sin x} \quad \text{or} \quad \sin x \csc x = 1$$

Pythagorean

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Quotient

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sec x}{\csc x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\csc x}{\sec x}$$

Odd-Even

$\sin(-x) = -\sin x$	(odd)	$\csc(-x) = -\csc x$	(odd)
$\cos(-x) = \cos x$	(even)	$\sec(-x) = \sec x$	(even)
$\tan(-x) = -\tan x$	(odd)	$\cot(-x) = -\cot x$	(odd)

Co-function

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Composite Argument (Sum and Difference)

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \qquad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Argument

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Half Argument

$$\sin\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1-\cos x)}$$

$$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1+\cos x)}$$

$$\tan\frac{1}{2}x = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

Sum and Product

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = -\cos(A+B) + \cos(A-B)$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$\cos x + \cos y = 2\cos\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)$$

$$\cos x - \cos y = -2\sin\frac{1}{2}(x+y)\sin\frac{1}{2}(x-y)$$

$$\sin x + \sin y = 2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)$$

$$\sin x - \sin y = 2\cos\frac{1}{2}(x+y)\sin\frac{1}{2}(x-y)$$

Linear Combination of Sine and Cosine

$$A\cos x + B\sin x = C\cos(x-D)$$

where $C = \sqrt{A^2+B^2}$ and $\cos D = \frac{A}{C}$ and $\sin D = \frac{B}{C}$