

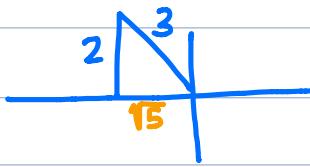
$$\textcircled{1} \quad \sin \theta = \frac{2}{3}$$

$$\frac{\pi}{2} < \theta < \pi$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{2}{3} \right) \left( -\frac{1}{3} \right)$$

$$= \boxed{-\frac{4\sqrt{5}}{9}}$$



$$x^2 + 2^2 = 3^2$$

$$x^2 = 5$$

$$\textcircled{2} \quad \cos \theta = \frac{1}{8} \quad (\text{positive})$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1}{2}(1-\cos \theta)}$$

$$= \sqrt{\frac{1}{2}(1-\frac{1}{8})}$$

$$= \sqrt{\frac{1}{2}(\frac{7}{8})}$$

$$= \sqrt{\frac{7}{16}} = \boxed{\frac{\sqrt{7}}{4}}$$

$$\textcircled{3} \quad \cos A = \frac{1}{3}$$

$$\tan \frac{1}{2}A = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \sqrt{\frac{1-\frac{1}{3}}{1+\frac{1}{3}}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= \sqrt{\frac{2}{3} \div \frac{4}{3}} = \sqrt{\frac{2}{3} \cdot \frac{3}{4}} = \sqrt{\frac{2}{4}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\textcircled{4} \quad \frac{\sin 2A}{2 \cos^2 A} = \frac{2 \sin A \cos A}{2 \cos A \cos A}$$

$$= \frac{\sin A}{\cos A} = \boxed{\tan A}$$

$$\textcircled{5} \quad \frac{\sin 2x}{2 \sin x} = \frac{2 \sin x \cos x}{2 \sin x}$$

$$= \boxed{\cos x}$$

$$\textcircled{6} \quad \frac{\sin 2A}{\cos A} - \sin A$$

$$\frac{2 \sin A \cos A}{\cos A} - \sin A$$

$$2 \sin A - \sin A = \boxed{\sin A}$$

$$\textcircled{7} \quad \sin A \cos A + \underline{\sin 2A}$$

$$\sin A \cos A + 2 \sin A \cos A$$

$$\boxed{3 \sin A \cos A}$$

$$\textcircled{8} \quad \frac{\sin 2x}{\sin(-x)}$$

$\leftarrow$  odd/even  $\sin(-x) = -\sin x$

$$\frac{2 \sin x \cos x}{-\sin x}$$

$$= \boxed{-2 \cos x}$$

$$\textcircled{9} \quad \sin \theta = \cos \theta \quad 0 \leq \theta < 2\pi$$

• ratios are only equivalent at  $\frac{\pi}{4}$  reference angle

• must be same sign (both positive or both negative)

$$\frac{\sin \theta}{\cos \theta}$$

$\boxed{2 \text{ solutions}}$

$$\textcircled{10} \quad \tan \theta = \frac{1+\sqrt{3}}{4}$$

positive ratio

$$\frac{\sin \theta}{\cos \theta}$$

$\boxed{\text{QI} \text{ } \& \text{ QIII}}$

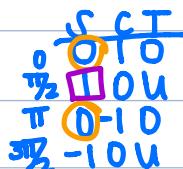
$$\textcircled{11} \quad \sin^2 x = \sin x \quad 0 \leq x < 2\pi$$

$$\boxed{0, \frac{\pi}{2}, \pi}$$

$$\sin^2 x - \sin x = 0$$

$$\sin x = 0$$

$$\sin x - 1 = 0$$



$$\sin x (\sin x - 1) = 0$$

$$\boxed{0 + 2\pi n, \pi + 2\pi n}$$

$$\sin^{-1}(0) = x$$

$$\sin^{-1}(1) = x$$

$$\boxed{\frac{\pi}{2} + 2\pi n}$$

$$(12) \quad 2\sin^2\theta + \sin\theta = 1 \quad 0 \leq \theta < 2\pi \quad \boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

$$2\sin^2\theta + \sin\theta - 1 = 0$$

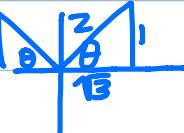
$$\cancel{2} \cancel{\sin^2\theta} (2\sin^2\theta + 2\sin\theta) \cancel{+ \sin\theta - 1} = 0$$

$$2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$$

$$(\sin\theta + 1)(2\sin\theta - 1) = 0$$

$$\sin\theta + 1 = 0$$

$$2\sin\theta - 1 = 0$$



$$\sin^{-1}(-1) = \theta$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \theta$$

$$\frac{3\pi}{2} + 2\pi n$$

$$\frac{\pi}{6} + 2\pi n$$

$$\frac{5\pi}{6} + 2\pi n$$

$$(13) \quad 2\cos^2\theta - 5\cos\theta - 3 = 0 \quad 0 \leq \theta < 2\pi$$

~~$$(2\cos^2\theta - 6\cos\theta) + 1(\cos\theta - 3) = 0$$~~

$$2\cos\theta(\cos\theta - 3) + 1(\cos\theta - 3) = 0$$

$$(\cos\theta - 3)(2\cos\theta + 1) = 0$$

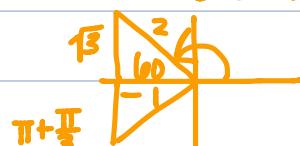
$$\cos\theta - 3 = 0$$

$$2\cos\theta + 1 = 0$$

$$\cos\theta \neq 3$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \theta$$

DNE



$$\frac{2\pi}{3} + 2\pi n$$

$$\frac{4\pi}{3} + 2\pi n$$

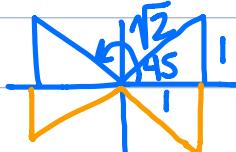
$$(14) \quad 2\sin^2\theta - 1 = 0$$

$$\frac{2\sin^2\theta}{2} = \frac{1}{2}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}}$$

$$\sin\theta = \pm \frac{1}{\sqrt{2}}$$

$$\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \theta$$



$$\boxed{\begin{array}{l} \textcircled{+} \quad \frac{\pi}{4} + 2\pi n \\ \textcircled{-} \quad \frac{3\pi}{4} + 2\pi n \\ \textcircled{-} \quad -\frac{\pi}{4} + 2\pi n \\ \textcircled{+} \quad \frac{5\pi}{4} + 2\pi n \end{array}}$$

$$\textcircled{15} \quad \csc^2 \theta - 2 \csc \theta = 0$$

$$\csc \theta (\csc \theta - 2) = 0$$

$$\csc \theta = 0$$

$$\csc^{-1}(0) = \theta$$

DNE

$$\csc \theta - 2 = 0$$

$$\csc^{-1}(2) = 0$$

$$\boxed{\frac{\pi}{6} + 2\pi n \quad \frac{5\pi}{6} + 2\pi n}$$



$$\csc \theta = \frac{\text{HYP}}{\text{OPP}}$$

$$\textcircled{16} \quad 2\cos^2 \theta + 7\cos \theta - 4 = 0$$

~~$$(2\cos^2 \theta + 8\cos \theta)(1\cos \theta - 4) = 0$$~~

$$2\cos \theta (\cos \theta + 4) - 1(\cos \theta + 4) = 0$$

$$(\cos \theta + 4)(2\cos \theta - 1) = 0$$

$$\cos \theta + 4 = 0$$

$$\cos \theta \neq -4$$

DNE

$$2\cos \theta - 1 = 0$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \theta$$

$$\boxed{\frac{\pi}{3} + 2\pi n \quad -\frac{\pi}{3} + 2\pi n}$$

$$\frac{\pi}{3}$$

$$\textcircled{17} \quad 5 + 4\cos \theta - 4\sin^2 \theta = 0 \quad \sin^2 x = 1 - \cos^2 x$$

$$5 + 4\cos \theta - 4(1 - \cos^2 \theta) = 0$$

$$4\cos^2 \theta + 4\cos \theta + 1 = 0$$

$$(2\cos \theta + 1)^2 = 0$$

$$2\cos \theta + 1 = 0$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \theta \quad \boxed{\frac{2\pi}{3} + 2\pi n}$$

$$\boxed{\frac{4\pi}{3} + 2\pi n}$$

$$\cancel{2} \quad (4\cos^2 \theta + 2\cos \theta)(2\cos \theta + 1) = 0$$

$$2\cos \theta (2\cos \theta + 1) + 1(2\cos \theta + 1) = 0$$

$$(2\cos \theta + 1)(2\cos \theta + 1) = 0$$

$$\textcircled{18} \quad 2\cos^2 \theta - \cos \theta = 0$$

$$\cos \theta (2\cos \theta - 1) = 0$$

$$\boxed{\frac{\pi}{3}}$$

$$\cos \theta = 0$$

$$2\cos \theta - 1 = 0$$

$$\cos^{-1}(0) = \theta$$

$$\cos^{-1}\left(\frac{1}{2}\right) = 0$$

$$\frac{\pi}{2} + 2\pi n$$

~~$$\frac{2\pi}{3} + 2\pi n$$~~

$$\boxed{\frac{\pi}{3} + 2\pi n}$$

$$\frac{3\pi}{2} + 2\pi n$$

$$-\frac{\pi}{3} + 2\pi n$$

19)  $2\sin^2\theta - 5\sin\theta - 3 = 0$

~~$(2\sin^2\theta - 6\sin\theta) + (\sin\theta - 3) = 0$~~

$$2\sin\theta(\sin\theta - 3) + 1(\sin\theta - 3) = 0$$

$$\sin\theta - 3 = 0 \quad 2\sin\theta + 1 = 0$$

$$\sin\theta \neq 3$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = \theta$$

DNE

~~$\begin{array}{c} S \\ T \\ C \end{array}$~~   $-30^\circ + 360n$

$$210^\circ + 360n$$

20)  $(\sin x + \cos x)^2 = 2$   $(\sin x + \cos x)(\sin x + \cos x) = \sin^2 x + 2\sin x \cos x + \cos^2 x$

$$\underline{\sin^2 x} + \underline{\cos^2 x} + 2\sin x \cos x = 2$$

$$1 + 2\sin x \cos x = 2$$

$$1 + \sin(2x) = 2$$

$$\sin(2x) = 1$$

$$\sin^{-1}(1) = 2x$$

$$\frac{1}{2}\left(\frac{\pi}{2} + 2\pi n\right) = \frac{2x}{2}$$

$$\frac{\pi}{4} + \pi n = x \quad \boxed{\frac{\pi}{4}} \quad \boxed{\frac{5\pi}{4}}$$

21)  $\cos 2\theta + \sin 2\theta = -1$

$$135^\circ$$

$$(2\cos^2\theta - 1) + 2\sin\theta \cos\theta + 1 = 0$$

$$2\cos\theta(\cos\theta + \sin\theta) = 0$$

$$\underline{2\cos\theta = 0}$$

$$\cos\theta + \sin\theta = 0$$

$$\cos^{-1}(0) = \theta$$

$$\cos\theta = -\sin\theta$$

$$90^\circ + 360n$$

see #9 but  
need opposite signs

$$\begin{array}{c} \sin+ \\ \cos- \end{array} \rightarrow \begin{array}{c} S \\ T \\ C \end{array} \leftarrow \begin{array}{c} \cos+ \\ \sin- \end{array}$$

$$270^\circ + 360n$$

$$135^\circ + 360n \neq 315^\circ + 360n$$

$$(22) f(x) = \frac{1}{1-\tan x} \quad \text{undefined}$$

denominator can't be zero!

$$\boxed{\frac{\pi}{4}}$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

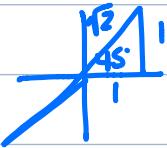
$$1 - \tan x = 0$$

$$-\tan x = -1$$

$$\tan x = 1$$

$$\tan^{-1}(1) = x$$

$$\frac{\pi}{4} + \pi n$$



$\tan x$  is also undefined  
at  $\frac{\pi}{2} \pm \frac{3\pi}{2}$

$$\begin{array}{c} 0 \ 1 \ 0 \\ | \quad | \quad | \\ 1 \ 0 \ 0 \\ 0 \ -1 \ 0 \\ -1 \ 0 \ 0 \end{array}$$

$$(23) \tan(x+30) = \text{undefined}$$

$\tan$  is undefined @  $90^\circ \pm 270^\circ$  (see 010 chart)

$$x+30=90$$

$$\boxed{x=60}$$